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For D-1(H) & D-1 (Subs.)

Flow of liquids - Viscosity

Viscosity: When a liquid flows slowly and steadily over a fixed horizontal surface, i.e. when the flow is streamline, the layer in contact with the fixed surface is stationary and the velocity of the layers increases with the distance from the fixed surface. Each layer tends to accelerate the layer below and ~~tends~~ to bring it to its own ~~level~~ velocity and tends to retard the layer above and slow it down to its own velocity.

This property of a liquid by virtue of which it opposes relative motion between its different layers is known as viscosity or internal friction of the liquid.

Coefficient of Viscosity: Newton showed that the backward dragging, or viscous force F, acting tangentially on any liquid layer

$$\cancel{F \propto A,\ F \propto v,\ r}$$

$$F \propto A$$
$$\propto -v$$
$$\propto \frac{1}{x}$$

(Negative sign of v indicates that force is opposite to velocity)

or $F \propto -\frac{Av}{x}$ i.e. $F = -\eta \frac{Av}{x}$

where η is a constant, depending upon the nature of the liquid.

Now, v/x may be put as dv/dx, called velocity gradient.

Hence, $F = -\eta . A \frac{dv}{dx}$

This is known as Newton's law of viscous flow in stream line motion.

If $A = 1m^2$ and $\frac{dv}{dx} = 1s^{-1}$

we have $F = \eta$

Thus the coefficient of viscosity of a liquid may be defined as the tangential force required per unit area to maintain a unit velocity gradient i.e. to maintain unit ~~velocity~~ relative velocity between two layers unit distance ~~apart~~ apart. If this tangential force be unity, the coefficient of viscosity is unity, and is called Poise.

Dimension of η, It is clear from the relation above

$$\eta = \frac{F}{A\frac{dv}{dx}}$$

So that the dimensions of η are those of

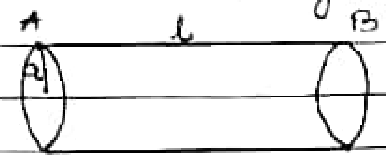
$$\frac{\text{force}}{\text{Area} \times \text{Velocity gradient}}$$

$$= \frac{MLT^{-2}}{[L^2]\left[\frac{L/T}{L}\right]} = \frac{MLT^{-2}}{L^2T^{-1}}$$

Or $\eta = ML^{-1}T^{-1}$

<u>Poiseuille's Eqn for flow of liquid through a tube.</u>

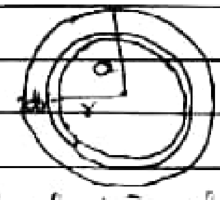
Let AB be a capillary tube of length l and radius a as shown in fig. If the flow of the liquid through it is steady, the velocity of the liquid along the walls is zero and is maximum along the axes of the tube



Now let consider a cylindrical layer of the liquid co-axial with the tube of inner radius r and outer radius $r+dr$. The velocity of the liquid at a distance r from the axes of the tube is v and at a distance $r+dr$ is $v-dv$, so that dv/dr is the velocity ~~grad~~ gradient

The surface area of the cylinder

$= 2\pi r l$



The liquid on the inner side of this cylindrical layer is moving faster while that on the outer side is moving slower

$\therefore$ Tangential force exerted by the outer layer on the inner layer opposite to the direction of motion

$$F = -\eta 2\pi r l \cdot \frac{dv}{dr}$$

The forward push due to the difference of pressure P

$$= P \times \pi r^2$$

When the flow is steady, there is no acceleration of the ~~fluid~~ liquid. ~~Hence~~

Hence, we have

$$-\eta 2\pi r l \frac{dv}{dr} = P\pi r^2$$

$$\text{or} \quad dv = \frac{-P}{2\eta l} \cdot r \cdot dr$$

Integrating both sides, we have

$$v = \frac{-P}{2\eta l} \times \frac{r^2}{2} + c$$

Where c is the const of integration

But velocity of the liquid along the axes the side of the tube is zero, therefore if $r = a$, $v = 0$

Substituting the values, we have

$$0 = -\frac{P}{2\eta l} \times \frac{a^2}{2} + c$$

$$\text{or} \quad c = \frac{P}{4\eta l} \cdot a^2$$

$$\text{Hence} \quad v = \frac{P}{4\eta l}\left[a^2 - r^2\right]$$

This is the average velocity of the liquid flowing through the cylindrical tube layer.

The area of cross-section of the cylindrical layer of radius r and thickness dr

$$= 2\pi r\, dr$$

Volume of liquid passing per sec through this area

$$dV = v \cdot 2\pi r\, dr$$

Hence the volume of liquid passing through the whole tube per sec is given by

$$\int dV = \int_0^a 2\pi v r\, dr$$

$$\text{or} \quad V = \int_0^a \frac{P}{4\eta l}(a^2 - r^2) \cdot 2\pi r\, dr$$

$$= \frac{\pi P}{2\eta l}\left[\frac{a^2 r^2}{2} - \frac{r^4}{4}\right]_0^a$$

$$= \frac{\pi P}{2\eta l}\left[\frac{a^4}{2} - \frac{a^4}{4}\right]$$

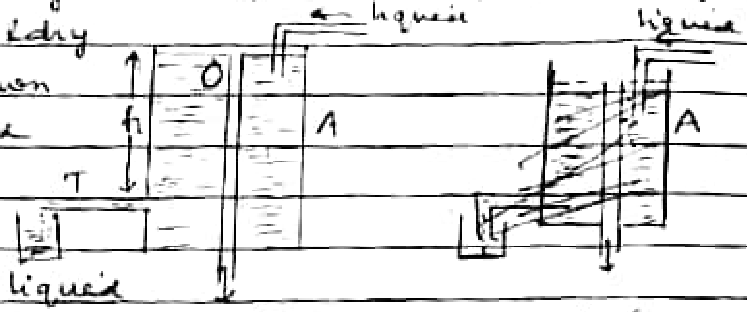
$$= \frac{\pi P a^4}{8\eta l}$$

$$\text{or} \quad \eta = \frac{\pi P a^4}{8Vl} \qquad \text{(A)}$$

Thus if we know P, a, l & V the coefficient of viscosity η can be determined.

Experimental determination of η for a liquid - Poiseuille's method

A capillary tube T of known length l and radius a is fixed horizontally near to the bottom of vessel A, the liquid level in which can be kept constant at any desired level by means of an over-flow arrangement O. A clean dry beaker of known weight is placed below the outlet end of the tube T to collect the liquid



flowing through it. The liquid is allowed to flow out in a slow trickle and collected in the beaker for a known time and the beaker is weighed again. The diff. gives the mass of liquid ~~flowed~~ flowing out in that time. Then, knowing the density of the liquid, its volume can be determined, and, dividing it by the time for which the liquid was allowed to flow, its volume flowing out per second is known. If h be the height of the liquid in the vessel above the axis of the tube, the difference of pressure $P = h\rho g$. The length of the ~~rod~~ tube is measured by a metre scale and radius with a high power microscope accurately. Substituting the values in the above eqn (A), the value of η can be determined.

Assumptions (i) The flow is steady and streamline

(ii) The pressure is constant over every section of the tube. i.e. there is no radial flow

(iii) The liquid in contact with the sides of the tube is stationary.

When the velocity of flow is small, and the tube is a narrow one, these assumptions are valid.

Streamline and Turbulent velocity
Bernoulli's Theorem

Streamline flow: If in a flow of liquid each particle ~~follows~~ follows exactly the same path and has the same velocity as its predecessor the ~~flow is streamline said to~~ liquid is said to have an orderly or a streamline flow. A stream line may also be defined as a curve, the tangent to which at any point gives the direction of flow of the liquid; for, ~~which~~ it may be straight or curved, according as the lateral pressure on it is the same throughout or different.

This holds good, however, only so long as the velocity of the liquid does not exceed a particular limiting value, called its critical velocity, beyond which the flow of the liquid loses all its steadiness or ~~and~~ orderliness and becomes zig zag or sinuous, ~~according~~ acquiring what is called a turbulent motion.

Bernoulli's Theorem: Bernoulli's theorem states that the total energy of a small amount of liquid flowing from one point to another, without any friction, remains constant through out the displacement. The energy of a liquid in motion at any point consists of the following three forms.

(i) Potential energy (ii) Kinetic energy (iii) Pressure energy. We know that the potential energy and pressure energy ~~are~~ of a liquid are convertible, one into another, and so are its pressure energy and kinetic energy. ~~It~~ It follows, therefore, that in any stream line flow of liquid, the loss of energy in one form is equal to the gain in energy in another, or that the sum total of its energy, i.e.

Potential energy + Pressure energy + Kinetic energy = C, a constant

or $\cancel{h + \frac{P}{\rho g} + \frac{1}{2}\frac{v^2}{g}}$ ~~kg~~

$= hg + \frac{P}{\rho} + \frac{1}{2}v^2 = C$ (Const)