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$\mathcal{D}_2$ (H)

Laws of Reflection & Refraction on the basis of E.M. Theory.

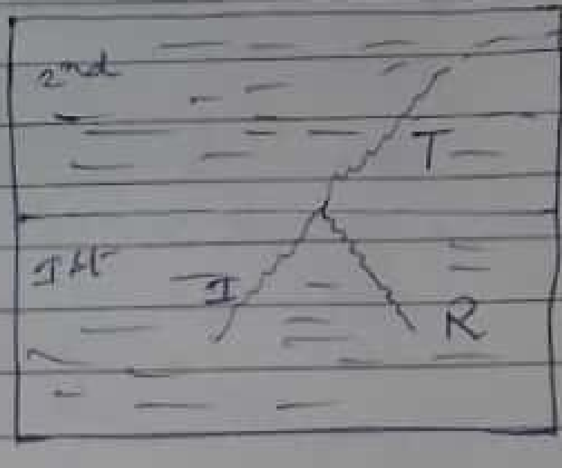
Q:- Discuss the laws of reflection & Refraction on the basis of E.M theory at the boundary separating two dielectrics.

Ans:- When a plane e.m. wave falls on to the boundary between two homogeneous dielectric media of different optical properties, it is split in to two waves:-

(i) a transmitted wave producing in to the second medium and

(ii) a reflected wave propagated back in to the first medium.

The existance of these two waves can be demonstrated from the boundary conditions. There are also plane waves (Shown in fig)



Now let us consider a plane polarised e.m. wave incident on an interface of two media, at an angle θ_i.

Y-Z plane (or $x=0$ plane) represents the plane of interface. Let ϵ_1, μ_1 and ϵ_2, μ_2 represents the dielectric permittivity and magnetic permeability of the media below & above the interface. We assume the two media to be less dielectrics.

Now let $\vec{E_i}$, $\vec{E_r}$ and $\vec{E_t}$ denotes the electric fields associated with the incident wave, refracted wave & ~~the~~ transmitted wave respectively and can be expressed as

$$\left.\begin{aligned} \vec{E}_i &= \vec{E}_{i_0} e^{i[\vec{K}_i \cdot \vec{r} - \omega_i t]} \\ \vec{E}_r &= \vec{E}_{r_0} e^{i[\vec{K}_r \cdot \vec{r} - \omega_r t]} \\ \vec{E}_t &= \vec{E}_{t_0} e^{i[\vec{K}_t \cdot \vec{r} - \omega_t t]} \end{aligned}\right\} \quad -(1)$$

where $\vec{E}_{i_0}$, $\vec{E}_{r_0}$ & $\vec{E}_{t_0}$ are the amplitude vectors and are independent of space & time but these may in general be complex. The vectors $\vec{k}_i$, $\vec{K}_r$ & $\vec{k}_t$ are the propagation vectors associated with the incident, refracted and transmitted waves respectively. We have the phase velocity

$$v = \frac{\omega}{K} = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\therefore \quad \left.\begin{aligned} \vec{K}_i^2 &= \omega_i^2 \mu_1 \epsilon_1 \\ \vec{K}_r^2 &= \omega_r^2 \mu_1 \epsilon_1 \\ K_t^2 &= \omega_t^2 \mu_2 \epsilon_2 \end{aligned}\right\} \quad -(2)$$

Now the tangential components of $\vec{E}$ must be continious across the interface. All the continuity conditions are to be satisfied at all times (ie space and time varying components of the phase are equal.) So, the coefficients of x, y, z & t in the exponents in eqⁿ (1) must be equal.

Thus we have

$$\omega_i = \omega_r = \omega_t = \omega \text{ (say)} \quad -(3)$$

This means frequency of the wave remains unchanged by reflection and refraction.

Therefore from eq^n (2) we have

$$k_i^2 = k_r^2 = \omega^2 \epsilon_1 \mu_1 \quad \text{— (3a)}$$
$$k_t^2 = \omega^2 \epsilon_2 \mu_2 \quad \text{— (3b)}$$

Further if k_{ix}, k_{iy}, k_{iz} represents the x, y & z components and similarly for k_r and k_t. Since the interface lies in y-z plane, so

$$\left.\begin{aligned} k_{iy} &= k_{ry} = k_{ty} = a \\ k_{iz} &= k_{rz} = k_{tz} = b \end{aligned}\right\} \text{— (4)}$$

Now as the wave is incident in x-z plane, then $k_{iy} = 0$.

It then follows that $k_{ry} = k_{ty} = 0$
This means that vectors $\vec{k}_{iy}$, $\vec{k}_r$ & $\vec{k}_t$ (i.e incident, reflected & transmitted waves) are in the same plane.

Further the z-components of the propagation vector are

$$k_i \sin\theta_i = k_t \sin\theta_t = k_r \sin\theta_r \quad \text{— (5)}$$

Since $\vec{k}_i = \vec{k}_r$, then we must have

$$\sin\theta_i = \sin\theta_r$$
$$\text{or} \quad \theta_i = \theta_r \quad \text{— (6)}$$

i.e. $\boxed{\text{angle of incidence = angle of reflection}}$

This explains the laws of reflection.

Further,

$$\frac{\sin\theta_i}{\sin\theta_r} = \frac{k_t}{k_i} = \frac{\omega\sqrt{\epsilon_2\mu_2}}{\omega\sqrt{\epsilon_1\mu_1}}$$

$$= \frac{\sqrt{\epsilon_2\mu_2}}{\sqrt{\epsilon_1\mu_1}}$$

$$= \frac{v_1}{v_2}$$

where $v_1 = \frac{1}{\sqrt{\mu_1\epsilon_1}}$ & $v_2 = \frac{1}{\sqrt{\epsilon_2\mu_2}}$

represents the speed of propagation of the waves in media (1) & (2), then

$$\frac{\sin\theta_i}{\sin\theta_t} = \frac{v_1}{v_2} = \frac{n_2}{n_1} = \frac{1}{n_2} \quad \text{—— (6b)}$$

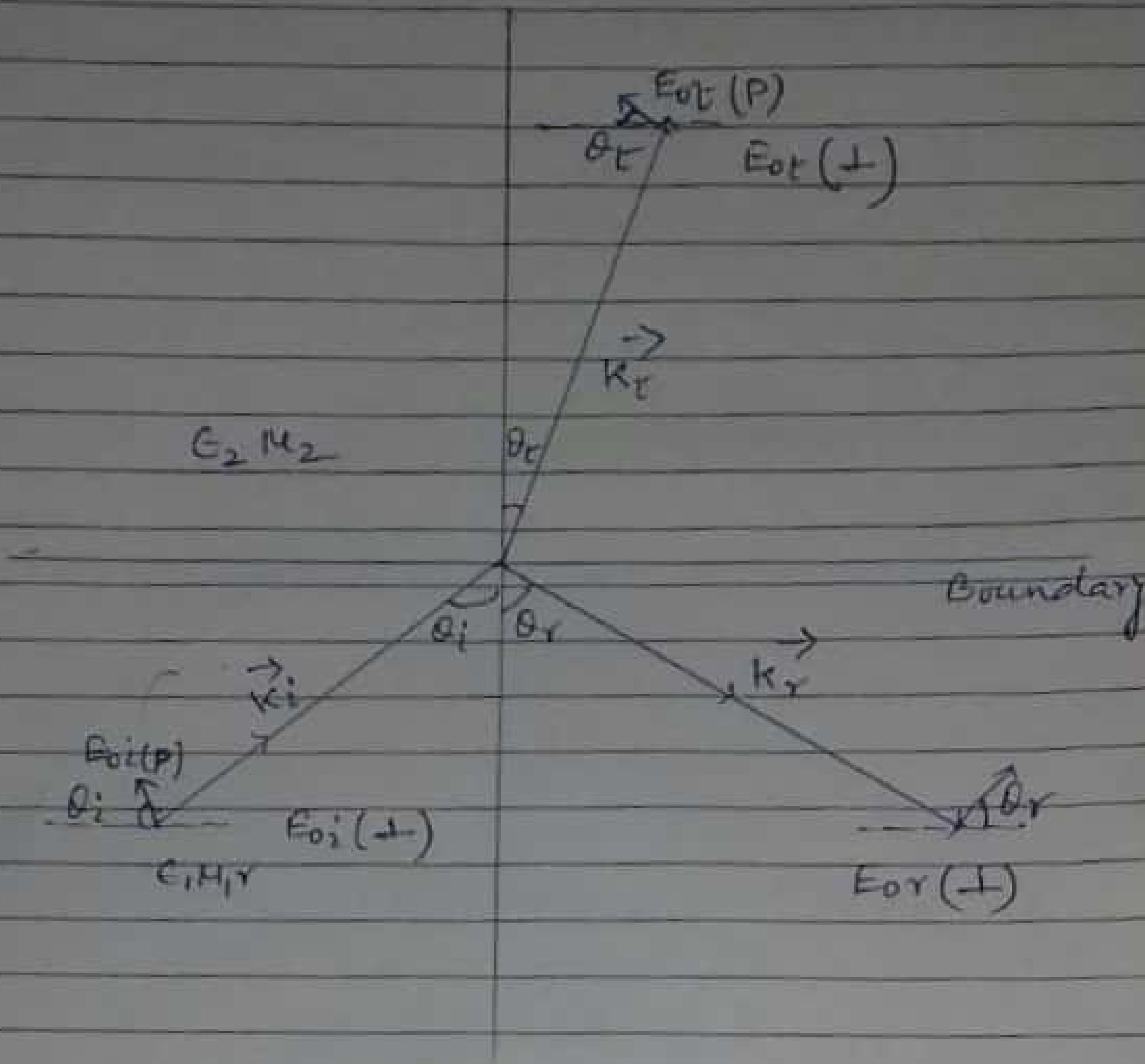
where $n_1 = \frac{c}{v_1}$ and $n_2 = \frac{c}{v_2} = c\sqrt{\epsilon_2\mu_2}$

$$= c\sqrt{\epsilon_1\mu_1} \quad \text{—— (7)}$$

represents the refractive indices of media (1) & (2). This is known as Snell's laws of refraction of E.M. waves.

Thus E.M. waves obey all the experimental laws of reflection & refraction at a surface separating two isotropic dielectric media.

Fig


E_{ot} (P)
θ_t
$E_{ot}(\perp)$
$\vec{K_t}$
$\epsilon_2 \mu_2$
θ_t
Boundary
θ_i
θ_r
$\vec{K_r}$
$\vec{K_i}$
$E_{oi}(P)$
θ_i
$E_{oi}(\perp)$
$\epsilon_1 \mu_1 r$
θ_r
$E_{or}(\perp)$