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Carnot's Engine and Carnot's cycle

Heat engines are practical devices which are used to convert heat energy into mechanical work. Sadi Carnot, a french engineer designed a theoretical heat engine which was free from all practical defects. The efficiency of Carnot's engine is maximum. It is therefore considered as an ideal engine. It consists of the following essential parts:

(i) A cylinder with perfectly non-conducting but a perfectly conducting bottom, filled with a perfectly non-conducting and frictionless piston. It contains air as working substance.

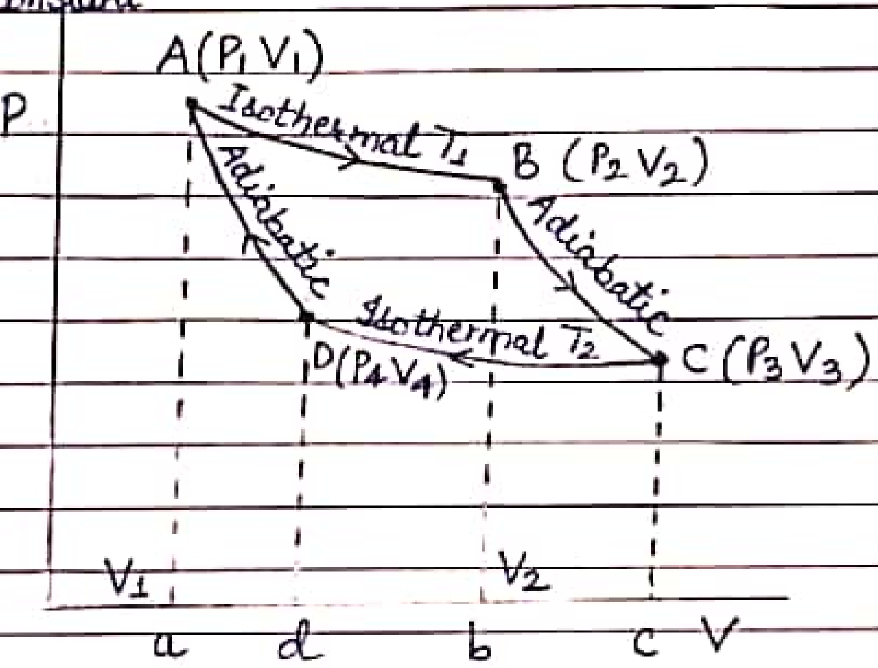
(ii) A hot reservoir or source of heat of infinite thermal capacity, maintained at a high and constant temperature, T_1.

(iii) A cold reservoir or sink, also of infinite thermal capacity, maintained at a lower but constant temperature T_2.

(iv) A perfectly non conducting stand, such that when desired, the cylinder may be moved to it without any friction.

In order to obtain a continuous supply of work, the working substance is subjected to the following cycle of quasi static operations, known as the Carnot cycle.

1.) The cylinder is first placed on the source, so that the gas acquires the temperature T_1 of the source, and the piston moved forward slowly. As the piston moves, the temperature tends to fall, and heat fan from the source to the cylinder. The operation is performed very slowly, so that the temperature of the air is always constant.



The representatine point on the indicator diagram moves from A to B, along the isothermal curve. The heat Q_1 extracted in this process is equal to the work done by the gas in this expansion, and is given by

$$W_1 = Q_1 = \int_{V_1}^{V_2} P\,dv = RT_1 \log \frac{V_2}{V_1}$$

$$= \text{area } AabB \quad \text{— (1.)}$$

2.) The cylinder is now removed from the source to the insulating stand, so that the gas is thermally isolated from the surrounding.

It is now allowed to undergo a slow adiabatic expansion, performing external work at the expense of internal energy, untill its temperature falls to T_2, the same as that of the sink. The operation is represented by the adiabatic BC. The work done W_2 by the gas is given by

$$W_2 = \int_{V_2}^{V_3} P dv = \frac{R(T_1 - T_2)}{\gamma - 1} = \text{area } BbcC \quad \text{---(2.)}$$

Since the pressure is now very much diminished, the gas has lost its expansive power, hence in order to enable it to recover its capacity for doing work it must be brought back to its original condition. To effect this the gas is compressed in two stages: first isothermally along the path CD and then adiabatically along DA. The point D is obtained by drawing the isothermal T_2 through C and the adiabatic through A.

3.) During the isothermal compression, the cylinder is placed in contact with the sink at T_2. The heat which is developed owing to compression will now pass to the sink. This is equal to work done on the gas and is equal to,

$$W_3 = Q_2 = \int_{V_4}^{V_3} P dv = RT_2 \log \frac{V_3}{V_4} = \text{area } CcdD \quad \text{---(3.)}$$

4) The cylinder is again removed to the insulating stand and the gas is compressed adiabatically. The work done on the gas by adiabatic compression is

$$W_4 = \int_{V_1}^{V_4} P\,dV = \frac{R}{\gamma-1}(T_1 - T_2) = \text{area } DdaA \quad \text{---} (4.)$$

It is thus seen that $W_2 = W_4$.

The net work done by the engine

$$W = W_1 + W_2 - W_3 - W_4 = \text{Area } ABCD$$

$$= W_1 - W_3 = Q_1 - Q_2 \quad \text{---} (5.)$$

Efficiency of the Engine: As we have seen, Q_1 is the heat absorbed by the gas during isothermal expansion and Q_2 is the heat rejected by it during isothermal compression, the rest $(Q_1 - Q_2)$ being converted into work.

The efficiency,

$$\eta = \frac{\text{Heat converted into work}}{\text{Heat absorbed from the source}}$$

$$= \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad \text{---} (6.)$$

Since points B and C lie on the same adiabatic BC, we have

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \quad \text{---} (7.)$$

Similarly, since D and A lie on the same adiabatic DA, we have

$$T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1} \text{ or } \frac{T_1}{T_2} = \left(\frac{V_4}{V_1}\right)^{\gamma-1} \quad \text{--- (8.)}$$

From (7.) and (8.),

$$\frac{V_3}{V_2} = \frac{V_4}{V_1} \text{ or } \frac{V_2}{V_1} = \frac{V_3}{V_4} \quad \text{--- (9.)}$$

$$\therefore \log \frac{V_2}{V_1} = \log \frac{V_3}{V_4} \quad \text{--- (10)}$$

$$\therefore \eta = \frac{R\left[T_1 - T_2\right]\log \frac{V_2}{V_1}}{R T_1 \log \frac{V_2}{V_1}} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1} \quad \text{--- (11.)}$$

From eq^n (6.) and (11.), we have

$$\therefore \frac{Q_1}{T_1} = \frac{Q_2}{T_2} \quad \text{--- (12.)}$$

It is thus clear that the efficiency of the engine depends upon temperature T_1 and T_2 of the source and the sink respectively and that the greater the value of $(T_1 - T_2)$, the higher the efficiency. Since, however $(T_1 - T_2)$ must always be less than T_1, it is clear that the efficiency must always be less than 1 or 100%.