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BETATRON [contd ---]

Calculation of the energy of the electrons :-

If the flux variation is given by the relation $\phi = \phi_0 \sin \omega t$, the time during which acceleration takes place will be $\pi/2\omega$. Hence the energy gained by the electron per turn when the flux changes will be

$$E = eV = e\frac{d\phi}{dt}$$

$$= e\frac{d}{dt}(\phi_0 \sin \omega t)$$

$$= e\phi_0 \omega \cos \omega t \quad \text{---(6)}$$

Thus, during the acceleration period its average value will be

$$E_{av} = \int_{t=0}^{t=\pi/2\omega} e\phi_0 \omega \cos \omega t = e\phi_0 \omega \left(\frac{1}{\omega}\right)_{0}^{\pi/2\omega}$$

$$= \frac{2}{\pi} e\omega\phi_0$$

---(7)

For most of the time electrons with travel with a velocity close to the velocity of light, therefore, the total distance traveled

during the acceleration process will be $c\pi/2\omega$. If R be the radius of the orbit, the no. of revolutions N will be given by

$$N = \frac{c}{4\omega R} \quad \text{---(8)}$$

Hence total energy gained by the electrons

$$E = E_{av} N$$

$$= \frac{2}{\pi} e\omega\phi_0 \times \frac{c}{4\omega R} = \frac{ec\phi_0}{2\pi R}$$

---(9)

The energy of electrons can also be estimated by the help of relativistic eqn for energy

$$K.E = pc = BeR \cdot c \quad \text{---(10)}$$

Since the operation of the betatron is unaffected by the increasing mass of the electron, as it gains energy it might be thought that there would be no upper limit of energy obtainable by this device. At a certain limit the energy gain is balanced by the radiation loss (because of the centripetal force, the circulating electrons are expected to emit radiations).

Schwinger suggested that the energy radiated per revolution was proportional to E^4/R. Thus

the radiation loss is very serious at very high energies and this device can not be used for accelerations of electrons to very large energies.

One method to minimise radiation loss is to increase the radius of the equilibrium orbit. Another method is to reduce the no. of revolutions, the electron makes between injection & ejection which can be done by using larger value of angular frequency (ω). This will not effect the final energy of the electron as ω does not enter in to the expression of the energy. Both of these measures have been adopted as far as possible in the 300 Mev betatron having 48" diameter. This is considered to largest useful size for a betatron.